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The dynamical gcd problem over function fields

A classic result of Bugeaud, Corvaja, and Zannier shows that if a and b are multiplicatively independent integers and $e > 0$, then $\gcd(a^n - 1, b^n - 1) < exp(en)$ for all sufficiently large n. Ailon and Rudnick later showed that something is even stronger over function fields in characteristic 0, namely that if a, b are nonconstant multiplicatively independent polynomials in $\mathbb{C}[x]$, then the degree of $\gcd(a^n - 1, b^n - 1)$ is bounded by a constant. We seek to treat this problem in a more general dynamical context. In particular, we are able to show that Ailon-Rudnick's result holds more generally for $\gcd(a^n - c, b^n - d)$ for any nonzero polynomials c, d. The method of proof is to replace the use of the Serre-Ihara-Tate theorem (which classifies plane curves containing infinitely many torsion points) in Ailon-Rudnick's paper with ideas of Baker-DeMarco-Rumely that allow one to treat more general families of points of small canonical height over arbitrary product formula fields. This represents joint work with Liang-Chung Hsia and Joe Silverman.